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Financial integration estimation with realized measures

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Financial integration estimation with realized measures

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Abstract

The objective of this study is to provide a new evidence on time-varying equity market integration, employing alternative econometric specifications of the conditional covariance process. Differently from the current literature on the topic, we specify alternative econometric models for the conditional covariance of stock indexes which include as a measure of past variability the monthly realized covariances. We analyze the degree of integration with the rest of the world of European equity markets and its variation through time. We cast our analysis in the framework provided with by the International Asset Pricing Model (IAPM). This model accommodates the evolving market structure from segmentation to integration as well as intermediate cases, depending on the existence of barriers to investments and the availability of substitute assets. Our analysis provides evidence that in recent years most of European Markets become more integrated with the world market. The local risk factor does not seem to be a determinant factor in the European markets, in the sample period considered. Its contribution to the total time-varying risk premium is only marginal.

1. Introduction

More financially integrated markets should lower the cost of capital, increase the investment opportunity set for local and foreigner investors and lead to significant welfare gains from higher savings and growth rate made possible by international risk sharing. The integration of equity markets is evolving in time and depends on structural reforms that affect not only the financial sector but the entire economy as well as. The evolution of market integration is also affected by the ability of foreign investors to access potentially segmentated markets as well as the ability of domestic investors to invest abroad, see Carrieri et al. (2007). In this report we follow an approach popular in the recent literature on financial integration based on a theoretical international asset pricing model. In this framework, the assets with identical risk should command for the same expected return in countries that are fully integrated. But this is not always the case, as described below. The specific imperfection relates to the assumed inability of a class of investors to trade in a subset of securities as a result of portfolio inflow restrictions imposed by some governments. The study of financial integration is strictly correlated with the literature dealing with contagion, systemic risk and the international portfolio optimization (see, e.g., Devereux and Yu (2014); Lehkonen (2015); Giglio et al. (2016); Rigobon (2016), for details).

The objective of this study is to provide a new evidence on time-varying equity market integration, employing alternative econometric specifications, based on the multivariate Generalized Autoregressive Conditional Heteroscedasticity (GARCH). We analyze the degree of integration with the rest of the world of European equity markets and its variation through time. We cast our analysis in the framework provided with by the International Asset Pricing Model (IAPM) of Errunza and Losq (1985). This model accommodates the evolving market structure from segmentation to integration as well as intermediate cases, depending on the existence of barriers to investments and the availability of substitute assets. Differently from the current literature on the topic we specify alternative econometric models for the conditional covariance of stock indexes which include as a measure of past variability the monthly realized covariances. We show that the estimated time-varying integration index is stable across the sample period with the exception of the financial crisis period. Our analysis provides evidence that in recent years most of European Markets become more integrated with the world market. The local risk factor, which is one of the factors determining the excess return in case of mild segmentation, does not seem to be a determinant factor in the European markets, in the sample period considered. Its contribution to the total time-varying risk premium (that is the financial compensation required by an investor to bear extra risk) is only marginal.

The paper is organized as follows. In Section 2, we review the literature on equilibrium models used for the estimation of financial integration. In Section 3, we develop the theoretical model while in Section 4 we introduce the econometric model. Results are presented in Section 5. Section 6 concludes.

2. Literature review

In this section, we provide a short review of the literature on financial market integration. We focus mainly on linear factor models introduced to provide an explanation of the financial integration based on the equilibrium approach.

2.1 Linear factor models for market integration

The literature distinguishes between full integration, complete segmentation, and the intermediate case of mild segmentation. Several equilibrium models, based on the assumption that there exists an equilibrium relationship between portfolio risk and expected return of assets, have been introduced to model and measure financial integration. The market is said to be *completely segmented* when the asset pricing restriction is country specific and the returns are only function of domestic risk factors. In this case, the portfolio allocation is constrained to domestic assets. The asset pricing theory applied to a single country suggests several models for completely segmented market. The workhorse model is the CAPM proposed by Sharpe (1964), Lintner (1965) and Black (1972). In this case, the market portfolio of country C, denoted by R_C, is the only systematic source of risk:

$$\begin{aligned} E[R_i^C] &= R_f^C + \beta_i(E[R_C] - R_f^C) \\ &= R_f^C + \lambda_C \operatorname{Cov}[R_i^C, R_C], \end{aligned} \tag{1}$$

where $E[R_i^C]$ is the expected return on asset i, R_f^C is the risk free rate in country C and β_i is the market loading of asset i. The domestic price of market risk in country C is $\lambda_C = (E[R_C] - R_f^C)/\operatorname{Var}(R_C)$. At a country level, we have $E[R_C] = R_f^C + \lambda_C \operatorname{Var}[R_C]$. Several extensions of the domestic CAPM has been considered. The Arbitrage Pricing Theory by Ross (1976) introduces several systematic sources of risk to explain the stock prices. For example, Merton (1987); Fama and French (1993, 2015); Acharya and Pedersen (2005) introduce additional risk factors in the specification of excess returns.

The second polar case is the full integration of the market. Market is perfectly integrated when the same asset pricing restriction holds in every country and the expected returns are function of only global risk factors. Investors can benefit from all international investment opportunities. This definition is in line with that provided by the report of the European Central Bank (2007). Solnik (1974, 1983); Grauer et al. (1976) provide the International CAPM based on the assumption that *Law of One price* holds, i.e. identical assets have the same price regardless of the country they are traded. For an asset i, the classical International CAPM states that:

$$\begin{aligned} E[R_i] &= R_f + \beta_i(E[R_W] - R_f) \\ &= R_f + \lambda_W \operatorname{Cov}[R_i, R_W], \end{aligned} \tag{2}$$

where R_W is the expected return on the world market and $\lambda_W = (E[R_W] - R_f)/\operatorname{Var}[R_W]$ is the world price of market risk. In this framework, the domestic risk is not rewarded because it is eliminated by the diversification. Harvey (1991); Dumas and Solnik (1995) provide a conditional framework allowing for time-varying market risk premium and time-variation in the rewards of exchange rate risk, respectively. However, the assumption of perfectly integrated market is too strong w.r.t. the empirical evidence (e.g., Jorion and Schwartz (1986); Karolyi and Stulz (2002) show the theoretical failures of the International CAPM).

Errunza (1992) test the hypothesis of full integration and complete segmentation for a group of emerging markets. The results provide strong evidence in favor of a mild segmentation structure. In a more general framework, Arouri et al. (2012) establish that if some investors do not hold all international assets because of direct and/or indirect barriers, the world market portfolio is not efficient and the traditional international CAPM must be augmented by a new factor reflecting the local risk undiversifiable internationally.

Bekaert and Harvey (1995) provide an extension of the static model by Errunza and Losq (1985), assuming that the degree of integration is variable over time. They propose a conditional regime switching model where countries are allowed to shift from segmentation to integration according transition probability. Most recent papers provide empirical assessment about the evolution of market integration showing that emerging markets are partially segmented, whereas developed markets are highly integrated into the world market (see, e.g, Bhattacharya and Daouk (2002); Adler and Qi (2003); Hardouvelis et al. (2006); Carrieri et al. (2007); Pukthuanthong and Roll (2009); Bali and Cakici (2010); Frijns et al. (2012)).

In the real world, the markets are partially integrated or mild segmented. Black (1974) put forward a model of capital market equilibrium with explicit barriers to international investment in the form of a tax on foreigner holdings of assets. Stulz (1981); Cooper and Kaplanis (2000) extend the Black's model showing that the tax level is the main variable that affects the portfolio asset allocation and the resulting market segmentation. A more general approach to deal with the mild segmentation of domestic markets is proposed in Errunza and Losq (1985). They consider a two-country capital market model. They assume that foreign (or unrestricted) investors can trade on both domestic and foreign assets, whereas the domestic (or restricted) investors can only invest in domestic assets. In this model, the authors show that the eligible assets (assets from the domestic country) are priced as in the classical International CAPM, see Section 3 below.

Considering the degree of market integration, Table 1 provides a summary of the models described above. The market integration is linked to the portfolio allocation problem. In Table 1 we highlight the determinants of excess returns, i.e. the systematic risk factors, and the portfolio composition for each degree of market integration. This should make clear how the markets price assets under different capital mobility regimes.

Alternatively to this literature, Chen and Knez (1995) propose a general arbitrage approach to define a test for integration, avoiding referring to a particular asset pricing model. They define a market perfectly integrated if the *Law of One price* in not violated and there are not arbitrage opportunities. In this context, markets are fully integrated if only stochastic discount factor model prices assets in every country. It follows that measures of market integration are developed using a general definition of stochastic discount factor (see also Flood and Rose (2005)).

Market integration	Eq.	Portfolio allocation	Systematic factors
Completely segmented	(1)	Only domestic assets	Local risk factors
Partially segmented	(3)	World market portfolio and domestic assets $\rightarrow$ *Diversification degree*	Global and local risk factors
Perfectly integrated	(2)	International portfolio $\rightarrow$ *Full diversification*	Global risk factor

Table 1: Degree of integration, assets in portfolio allocation and systemic factors.

2.2 Financial integration indicators

In order to measure the financial integration degree, the literature proposes several indicators. A large part of the literature exploit the idea that the co-movements of stock market prices/returns are indicators of integration (see, e.g. King et al. (1994); Lin et al. (1994); Longin and Solnik (2001); Kearney and Lucey (2004)). The easiest way to measure co-movements is to calculate correlations between prices or returns, i.e. compute the Pearson's correlation coefficient (PCC) $\rho_{i,j} = \mathrm{Cov}[R_i, R_j]/\sqrt{(\mathrm{Var}[R_i]\,\mathrm{Var}[R_j])}$.

Brooks and Del Negro (2004); Candelon et al. (2008); Lucey and Zhang (2009), among others, show that the degree of co-movements is not constant over time and it is increasing during the last decades of years. Based on this issue, Brooks and Del Negro (2004) provide an average correlation indicator estimated through a rolling windows, i.e. $\bar{\rho} = 2/(N(N-1))\sum_i^N \sum_{j=i+1}^N \rho_{i,j}$ where $\rho_{i,j}$ is computed at the end of period selected. King and Wadhwani (1990) compute a similar correlation indicator on non-overlapping sample periods.

However, Carrieri et al. (2007); Pukthuanthong and Roll (2009); Billio et al. (2015) shows that the correlation coefficient tends to underestimate the integration degree. Furthermore, Forbes and Rigobon (2002) state that the integration degree is overestimated during crises due to the higher volatility on the stock market. Their correlation coefficient ρ_{FR_t} contains a correction term $\delta_{i,j}$ that accounts for the increase in the variance of returns, i.e. $\rho_{FR_{i,j}} = \rho_{i,j}/\sqrt{1+\delta_{i,j}(1-\rho_{i,j})}$. An interesting indicator for investigating co-movements of stocks returns that takes into account long term trends and short term fluctuations is proposed by Grauer et al. (1976). The indicator is based on wavelet analysis, a technique for decomposing a signal into frequencies (see also Rua and Nunes (2009)). Carrieri et al. (2007) link to the integration index by Errunza and Losq (1985) that is a function of the variances of R_I. Pukthuanthong and Roll (2009) propose to identify a set of common factors that can be interpreted as integration drivers (see also Berger et al. (2011); Berger and Pukthuanthong (2012)). The cross-country average adjusted R-square, estimated for each year by the Principal Component Analysis (PCA), is an alternative integration measure.

3. Theoretical framework

In this section, we provide the description of our theoretical model. We link to the model proposed by Errunza and Losq (1985). We consider the mild segmentation of capital markets model introduced in Errunza and Losq (1985). We introduce the following assumptions:

1. *Unequal Access Assumption*. The investing population is divided in two subsets: (i) the unrestricted investors can trade in all the securities available in the market; (ii) the restricted investors can trade only in a subset of the securities, the so-called *eligible* (*e*) . The noneligible or *ineligible* (*i*) securities can be held only by the unrestricted investors.
2. *Perfect Capital Market Assumption*. The capital markets are perfect and frictionless. This assumption includes equal access to information by all market participants, completely rational economic actors, and no transaction costs.
3. *Mean-Variance Assumption.* The expected utility of an investor is function of the expected value of returns and its variance.
4. *Free Lending and Borrowing Assumption.* Investors can borrow or lend any amount of money at the same risk-free rate of return.

Let us focus on two countries. In country 1 investors are restricted while in country 2 investors are unrestricted. Country 1 securities are eligible, on the contrary, country 2 securities are ineligible for country 1 investors. Specifically, *portfolio inflow restrictions* imposed by the government of country 2 prevent country 1 investors from holding country 2 securities; whereas no such controls are imposed by the government of country 1.

Country 1	Country 2
Restricted investors	Unrestricted investors
Eligible assets	Ineligible assets
Investors can trade only in a subset of the securities.	Investors can trade in all the securities available.

Table 2: Representation of the mild segmentation model.

Let us define the vector of returns $\mathbf{R} = [\mathbf{R}_i', \mathbf{R}_e']'$, where $\mathbf{R}_i$ and $\mathbf{R}_e$ are the vector of returns on the ineligible and eligible securities, respectively. The returns are supposed to be normally distributed [1] with covariance matrix

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_{ii} & \mathbf{\Sigma}_{ie} \\ \mathbf{\Sigma}_{ei} & \mathbf{\Sigma}_{ee} \end{bmatrix}.$$

In a similar way, the vector of aggregate market values is $\mathbf{P} = [\mathbf{P}_i', \mathbf{P}_e']'$.

To account for the partial integration of the market, we introduce the following portfolios (and their corresponding notation):

1. the World Market Portfolio: market value M, rate of return R_W, representative vector $WMP = \mathbf{P} = [\mathbf{P}_i', \mathbf{P}_e']'$;
2. the Market Portfolio of Ineligible Securities: market value M_I, rate of return $R_I = \mathbf{R}_i'\mathbf{P}_i$, representative vector $MPIS = [\mathbf{P}_i', \mathbf{0}']'$.;
3. the Market Portfolio of Eligible Securities: market value M_E, rate of return $R_E = \mathbf{R}_e'\mathbf{P}_e$, representative vector $MPES = [\mathbf{0}', \mathbf{P}_e']'$.

Since, the portfolio of eligible assets $\boldsymbol{R_e}$ is not observable, we estimate the diversified portfolio DP, that is the portfolio most hightly correlated with the market portfolio of ineligible assets R_I. Errunza et al. (1999) estimate DP from the set of industry portfolios. Carrieri et al. (2007) consider also the country funds (CF) and the American Depository

[1] In continuous time, the instantaneous returns are supposed to evolve according to a stationary diffusion process, with lognormal distributed prices.

Receipts (ADRs) to generate highly correlated return with the market portfolios of their ineligible assets. Given the assumption of normality, DP is the portfolio that minimize $\mathrm{Var}[R_I - \alpha'\mathbf{R}_e]$ w.r.t. to α, i.e., the optimal is $\alpha^* = \mathbf{\Sigma}_{ee}^{-1}\mathbf{\Sigma}_{Ie}$, where $\Sigma_{Ie} = \mathrm{Cov}[R_I, \mathbf{R}_e]$ Since $R_I = \mathbf{R}_i'\mathbf{P}_i$, $\Sigma_{Ie} = \mathrm{Cov}[\mathbf{R}_i'\mathbf{P}_i, \mathbf{R}_e] = \mathbf{\Sigma}_{ei}\mathbf{P}_i$ follows. The dollar amounts invested in the various securities are given by

$$DP = \begin{bmatrix} \mathbf{\Sigma}_{ee}^{-1}\mathbf{\Sigma}_{ei}\mathbf{P}_i \\ \mathbf{0} \end{bmatrix}.$$

Thus, the restricted investors can duplicate returns on unavailable assets through home-made diversification.

Errunza and Losq (1985) show that under market segmentation, the expected return on the *i*-th ineligible security in the I-th market is

$$\begin{aligned} E[R_i] &= R_f + AM\,\mathrm{Cov}\,[R_i, R_W] + (A_u - A)M_I\,\mathrm{Cov}\,[R_i, R_I|\mathbf{R}_e] \\ &= R_f + \lambda_W\,\mathrm{Cov}\,[R_i, R_W] + \lambda_I\,\mathrm{Cov}\,[R_i, R_I|\mathbf{R}_e], \end{aligned} \tag{3}$$

where asset *i* in the I-th market is accessible only to nationals. A is the aggregate risk aversion coefficient with $A^{-1} \equiv A_r^{-1} + A_u^{-1}$. A_u is the absolute risk aversion coefficient for unrestricted investors on the I-th market and A_r is the absolute risk aversion coefficient for restricted investors. The prices of risk λ_W and λ_I are functions of the relative risk aversions of restricted and unrestricted investors, as showed in Errunza and Losq (1985). The expected return on the potentially segmented market is proportional to the covariance with a global factor and to the conditional market risk[2].

The expected return on the ineligible security market index can be obtained aggregating over the ineligible set of securities:

$$E[R_I - R_f] = AM\,\mathrm{Cov}\,[R_i, R_W] + (A_u - A)M_I\,\mathrm{Var}\,[R_I|\mathbf{R}_e].$$

Under the assumption that the returns are jointly normally distributed, we have

$$\begin{aligned} \mathrm{Var}[R_I|\mathbf{R}_e] &= \mathrm{Var}[R_I] - \mathrm{Cov}[R_I, \mathbf{R}_e]'\,\mathrm{Var}[\mathbf{R}_e]^{-1}\,\mathrm{Cov}[R_I, \mathbf{R}_e] \\ &= \mathrm{Var}[R_I]\{1 - \rho^2(R_I, \mathbf{R}_e)\}, \end{aligned} \tag{4}$$

and

$$\rho^2(R_I, \mathbf{R}_e) = \frac{\mathrm{Cov}[R_I, \mathbf{R}_e]'\,\mathrm{Var}[\mathbf{R}_e]^{-1}\,\mathrm{Cov}[R_I, \mathbf{R}_e]}{\mathrm{Var}[R_I]},$$

where ρ is the multiple correlation coefficient that can be interpreted as the correlation coefficient between R_I and that portfolio of eligible securities which is most correlated with R_I, i.e., the DP portfolio. When $\rho = 0$ the extreme form of market segmentation takes place, i.e. when no correlation exists between R_I and the return on any eligible security and the market are completely segmented:

$$E[R_i] - R_f = A_u M_I\,\mathrm{Cov}\,[R_i, R_I].$$

The super risk premium in Eq. (3) vanishes in the following two limiting cases:

1. when the unrestricted investor becomes much less risk averse than the restricted one, i.e., the ratio A_r/A_u goes to ∞. [3]

2. when $\rho \to 1$, i.e., the return on DP is perfectly correlated with R_I, then the conditional market risk becomes negligible for all securities.

Errunza and Losq (1985) propose the following aggregate measure of substitution (integration):

$$II = 1 - \frac{\mathrm{Var}[R_I|\mathbf{R}_e]}{\mathrm{Var}[R_I]} = \rho^2(R_I, \mathbf{R}_e). \tag{5}$$

The integration index II ranges between 0 and 1, by definition. The extreme values correspond to the two polar cases of integration degree. In particular, when $II = 1$, i.e.

[2]The conditional market risk is defined as the conditional covariance between the return of asset i and the return on the market portfolio of all ineligible securities I, given the returns on all eligible securities. The conditional market risk can be interpreted as a measure of substitutability between a specific ineligible security and the eligible segment of the world market.

[3] In this case, we have (i $\mu \equiv (1 + A_r/A_u)^{-1} \to 0$, the unrestricted investor tends to hold the risky securities, i.e., he holds the world market portfolio; (ii) $(A_u - A)/A \equiv A_u/A_r \to 0$, the super risk premium becomes negligible.

$\text{Var}[R_I|\mathbf{R}_e] = 0$, the markets are fully integrated. In such a case, there exists a portfolio of eligible securities that is perfectly correlated with the return on market portfolio of ineligible securities (see Eq. (2)). The two segment of the market would be effectively integrated. The only measure of risk would be the beta coefficient defined relative to the world market portfolio. When $II = 0$, i.e. $\text{Var}[R_I|\mathbf{R}_e] = \text{Var}[R_I]$, the markets are completely segmented. The following table summarizes the interpretation of the integration index II.

$II = 0$	$\text{Var}[R_I\|\mathbf{R}_e] = \text{Var}[R_I]$	Completely segmented market
$0 < II < 1$	$\text{Var}[R_I\|\mathbf{R}_e] > 0$	Partially segmented market
$II = 1$	$\text{Var}[R_I\|\mathbf{R}_e] = 0$	Perfectly integrated market

Table 3: Interpretation of the financial integration index.

4. Econometric specification

Let use define $r_{*,t}, * = \{I, DP, W\}$ the excess return on the $R_{*,t}$ return index. From the Errunza and Losq (1985) model, the following system of equations must hold at any point in time,

$$\begin{cases} E_{t-1}[r_{I,t}] & = & \lambda_{W,t-1}\,\text{Cov}_{t-1}[r_{I,t}, r_{W,t}] + \lambda_{I,t-1}\,\text{Var}_{t-1}[r_{I,t}|r_{DP,t}] \\ E_{t-1}[r_{DP,t}] & = & \lambda_{W,t-1}\,\text{Cov}_{t-1}[r_{DP,t}, r_{W,t}] \\ E_{t-1}[r_{W,t}] & = & \lambda_{W,t-1}\,\text{Var}_{t-1}[r_{W,t}]. \end{cases} \tag{6}$$

The first equation in the system is the pricing of the local market index, where two factors are priced: the world market covariance risk and the super risk premium, proportional to the conditional local risk represented by $\text{Var}_{t-1}[r_{I,t}|r_{DP,t}]$. The second equation prices the DP through the covariance risk with the world portfolio return. Finally, the last equation is the pricing equation for the world index portfolio. The theory predicts that the world price of risk should be the same for each country.

The time-varying integration index II_t is:

$$II_t = \rho_t^2(r_{I,t}, r_{DP,t}) = \frac{\text{Cov}_{t-1}[r_{I,t}, r_{DP,t}]^2}{\text{Var}_{t-1}[r_{DP,t}]\,\text{Var}_{t-1}[r_{I,t}]},$$

where, according to Eq. (4), the conditional variance $\text{Var}_{t-1}[r_{I,t}|r_{DP,t}]$ can be written as

$$\text{Var}_{t-1}[r_{I,t}|r_{DP,t}] = \text{Var}_{t-1}[r_{I,t}](1 - \rho_{t-1}^2(I, DP)).$$

It follows that

$$\rho_t^2(I, DP) = \frac{\text{Cov}_{t-1}[r_{I,t}, r_{DP,t}]^2}{\text{Var}_{t-1}[r_{I,t}]\,\text{Var}_{t-1}[r_{DP,t}]}.$$

In matrix notation, we can rewrite Eq. (6) as follows:

$$\begin{bmatrix} r_{I,t} \\ r_{DP,t} \\ r_{W,t} \end{bmatrix} = \begin{bmatrix} \lambda_{W,t-1} & \lambda_{I,t-1} & 0 & 0 \\ 0 & 0 & \lambda_{W,t-1} & 0 \\ 0 & 0 & 0 & \lambda_{W,t-1} \end{bmatrix} \begin{bmatrix} H_{I,W,t} \\ H_{I,t}(1-\rho_t^2(I, DP)) \\ H_{DP,W,t} \\ H_{W,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{I,t} \\ \varepsilon_{DP,t} \\ \varepsilon_{W,t} \end{bmatrix} \tag{7}$$

$$r_t = \Lambda_{t-1} F_t + \varepsilon_t,$$

where the vector F_t contains functions of the elements of the conditional variance of $r_t = [r_{W,t}, r_{DP,t}, r_{I,t}]'$, denoted by H_t, and $\varepsilon_t = [\varepsilon_{I,t}, \varepsilon_{DP,t}, \varepsilon_{W,t}]'$ is the vector of error terms. The model needs the specification of the law of motion of the conditional covariance matrix, H_t. To this purpose we consider alternative specifications in the multivariate GARCH family, see Bauwens et al. (2006). In this case, the model in Eq. (7) is a GARCH-in-mean, where functions of the matrix H_t enter in the conditional mean. In this case the conditional information matrix is not diagonal which in turn implies that we have efficiency gains in jointly estimating all the parameters in the model, i.e. those in the conditional mean and covariance processes.

The GARCH model are usually appropriate for modeling conditional variances and covariances for stock market. Assuming a conditional Gaussian distribution of stock returns, the GARCH models allow components of variances and covariances vary over time depending on the shocks at time $t-1$ and on the past values of variances and covariances terms (see Bollerslev (1986); Engle (1982)). The model's parameters are estimated by Quasi Maximun Likelihood (QMLE), see Bollerslev and Wooldridge (1992). The QML estimator maximizes the log-likelihood function over the parameters in the specification of $\lambda_{W,t-1}$, $\lambda_{I,t-1}$, and the covariance matrix H of the errors. Since the number of parameters to estimate a GARCH model increases w.r.t. the number of variables involved, several constrained models have been introduced. The most popular specifications are the Costant Condtional Correlation (CCC) model by Bollerslev (1990) and the Baba-Engle-Kraft-Kroner (BEKK) model defined in Engle and Kroner (1995). Carrieri et al. (2007); Arouri et al. (2012) provide empirical results on the evolution of market integration by assuming a GARCH-in-mean methodology (see Bekaert and Wu (2000)). Boubakri et al. (2016) analyse the effect of financial crises on the international financial integration between emerging markets by using a Dynamic Conditional Correlation (DCC) model proposed by Engle (2002b). A similar model is used by Alotaibi and Mishra (2016) to study the financial integration for the region of the Gulf Cooperation Council (GCC).

Another issue concerns the property of the prices of risk λ_W and λ_I. Merton (1980); Adler and Dumas (1983) show that the world price of risk λ_W is a positive function of the risk aversion coefficient. Moreover, Harvey (1991); De Santis and Gerard (1997) state that the prices of risk are time-varying. Following these considerations, Bekaert and Harvey (1995); Carrieri et al. (2007); De Santis and Hillion (2003); Hardouvelis et al. (2006), among others, assume dynamic functions for the prices of risk λ_W and λ_I by using global and local factors. Thus, we assume that the evolution in time is assumed to be driven by a set of information variables, i.e.,

$$\lambda_{*,t-1} = \exp\left(\gamma_*' Z_{*,t-1}\right), \text{with } * = W, I,$$

where $Z_{*,t-1}$ is the set of global or local information variables available at time $t-1$, γ_* denote the vector of coefficients associated with these variables, as in Carrieri et al. (2007). The exponential specification ensures that the prices are positive.

5. Empirical application

In this section, we analyze the time-varying integration of European stock markets with respect to the world market. We estimate model in Eq. (7). This returns the estimate of II_t and the implied risk premia, that measures the financial integration in terms of global and local factors. The numerical computations are performed with MATLAB©.

5.1 Data

Our study focuses on a set of European countries: Austria (AT), Belgium (BE), Denmark (DK), Finland (FI), France (FR), Germany (GE), Ireland (IE), Italy (IT), Netherlands (NL), Norway (NO), Portugal (PT), Spain (ES), Sweden (SE), United Kingdom (UK). [4] Our data set includes the following groups of data:

1. daily data on returns of European and World market indexes (MSCI) used to compute the realized covariances (see Appendix 2);
2. monthly data on returns of European stock market and World market indexes. The World market portfolio is proxied by the MSCI value-weighted world index whereas the European stock market returns are computed from the MSCI indexes for each country;
3. monthly data on macroeconomic and financial variables likely to explain the prices of risks and used to construct the diversification portfolio: the monthly returns of 11 MSCI industry portfolios (see Table 4), the default spread (Moody's BAA-AAA bond yields) and the 30-day Eurodollar rate.

The data cover the period from January 1995 to August 2016. The monthly returns are defined as $R_{i,t} = \log(P_{i,t}/P_{i,t-1})$. The monthly excess return of each index is calculated using the one-month Eurodollar rate as a proxy of the risk free rate. Table 6 reports the summary

[4]Other EU countries are not considerate in the empirical application because the data are not available.

statistics for the monthly excess returns of European countries and the World market index. To analyze these data, we consider two subsamples: from January 1995 to July 2007, the so-called pre-crises subsample, and from August 2007 to August 2016. The European returns on average are positive and large in the pre-crises subsample. The returns display high volatility in the second subsample, as expected. In the full sample, the difference between the two subsamples are mitigate. The data for Austria and Belgium show a high level of kurtosis and normality test are rejected. The normality tests are not rejected for Italy. We also provide results for the Engle's ARCH test for heteroskedasticity. For most of the countries, this test is rejected. Table 6 also provides the descriptive statistic for the World market index and the correlation index between data on European stock market (r_I) and the world index (r_W). On average, the correlation index $\mathrm{Corr}(r_I, r_w)$ is 0.70, thus the data are positive correlated.

5.2 Diversification portfolio

The *Diversication portfolio* (DP) is the most highly correlated portfolio with the market portfolio of ineligible securities. In order to get an estimates for it, we regress $R_{I,t}$ on the MSCI world index and the MSCI global industry portfolios. The regressors included in the estimation procedure are listed in Table 4. In order to choose the better combination of

x_1	MSCI world index,
x_2	energy index,
x_3	materials index,
x_4	industrials index,
x_5	consumer discretionary index,
x_6	consumer staples index,
x_7	health care index,
x_8	financials index,
x_9	information technology index,
x_{10}	telecommunication services index,
x_{11}	utilities index,
x_{12}	real estate index.

Table 4: Variables in the stepwise regressions to determine the diversification portfolio for each country.

regressors to estimate the diversification portfolio, we apply the stepwise procedure. This methodology is based on a forward and backward threshold p-values. At each step, regressors are added if their p-values are less than α and are removed if their p-values are larger than $\alpha + 0.05$. Tables 7-9 report the estimated results for three different thresholded criteria with (i) $\alpha = 0.05$, (ii) $\alpha = 0.10$ and (iii) $\alpha = 0.15$. As expected, the number of regressors increases when we consider a large thresholded criteria (see e.g., the number of increases from 4 to 6 for Italy). However, between models (i)-(iii) there is not a large difference in term of coefficient of determination ($\bar{R}^2$). Thus, we select for our empirical application, the diversification portfolios constructed from the stepwise procedure with $\alpha = 0.10$. In this way, the diversification portfolios have been constructed ex post. That is, the construction of portfolios is based on information that would not have been available to market participants. In Table 6, for each country, we also report the correlation index between the diversified portfolio and market portfolio of ineligible securities (i.e., $\mathrm{Corr}(r_I, r_D)$). Indeed, the DIV portfolio is high and positive correlated with the corresponding excess returns index. We also observe the diversified portfolios are highly correlated with the world index (i.e., $\mathrm{Corr}(r_I, r_D)$ is always larger than 0.80).

5.3 Integration index and risk premium

The general theoretical model described in Section 3, allows us to built several representation that differ with respect to the conditional covariance model used (see Appendix 1), the assumption on prices of risk (i.e., time-invariant λ_* or time-varying $\lambda_{*,t}$) and the frequently of data involved in the computation. Table 5 give us the model taxonomy that arises from our theoretical framework and the availability of empirical data. We consider two conditional covariance models: the GARCH(1,1) model and GARCH models with cross-sectional market volatility. The GARCH(1,1) model involves the true conditional covariance matrix by mean zero errors in its parameterization. This model is the most used in the literature. On the opposite, the GARCH(1,1)-X model involves in the computation an estimate of the matrix of quadratic covariations based on the monthly realized variances and covariances.[5]

	GARCH(1,1)					
Model	Representation		Price of risk		Frequency	
	Complete	Diagonal	λ_*	$\lambda_{*,t}$	Daily	Monthly
1		x	x		x	
2	x		x		x	
3		x	x			x
4	x		x			x
5		x		x		x
6	x			x		x
	GARCH(1,1)-X					
Model	Representation		Price of risk		Frequency	
	Complete	Diagonal	λ_*	$\lambda_{*,t}$	Daily	Monthly
7		x	x			x
8	x		x			x
9		x		x		x
10	x			x		x

Table 5: Model taxonomy

Hereafter, for each European country we provide results for models that employ monthly data[6]. First, we estimate the models (3)-(10) for each country. Then, we select the better model through Akaike information criterion (AIC) and Bayesian information criterion (BIC) that measure of the relative quality of statistical models. Finally, we provide the estimates of time-varying integration degree (II_t) for the selected model. We can also assess the economic importance of the premium associated with the local risk factors by decomposing the total premium into the global and the local premium, GP and LP, respectively. Accordingly to the theoretical model, fluctuations in risk premia come from three different sources of variation: the price of risk, the degree of segmentation measured by $\mathrm{Var}_{t-1}[r_{I,t}|r_{DP,t}]$ and the covariance moments $\mathrm{Cov}_{t-1}[r_{I,t}, r_{W,t}]$. It worth mentioning that complete GARCH models include a far larger number of parameters to be estimated. This could pose numerical problems in terms of convergence of numerical algorithms used to maximize the loglikelihood function.

- *Estimation results for models with time-invariant prices of risk.*
  For each European country in the sample, we estimate models (3)-(4) using GARCH(1,1) and models (7)-(8) using GARCH-X representation. In order to select the model that provides the best estimate of the time-varying integration index II_t, we calculate the BIC and AIC, see Panel A of Table 10. For each country we identify the model that minimizes both criteria. It turns out that for all countries, the best trade-off, as represented by the information criteria, is the one provided by the complete specification. The GARCH(1,1)-X model, that includes the past realized covariances, is the model most frequently chosen by both criteria. In this setting, the prices of risk are assumed to be time-invariant, thus for each selected model the estimates of time-varying risk-premium is written as

$$\delta_{1,t} = GP_{1,t} + LP_{1,t} = \lambda_W \,\mathrm{Cov}_{t-1}[r_{I,t}, r_{W,t}] + \lambda_I \,\mathrm{Var}_{t-1}[r_{I,t}|r_{DP,t}]. \quad (8)$$

- *Estimation results for models with time-varying prices of risk.*
  Passing to specifications with time-varying prices of risk increases the number of parameter to estimate. For each European country in the sample, we estimate the models (5)-(6) using GARCH(1,1) and models (9)-(10) using GARCH-X representation. In

[5]Technical details on the parametrization of conditional covariance models and estimation of realized covariances are reported in Appendices 1 and 2.

[6]For robustness purposes the GARCH models have been estimated also with daily data. The results are available upon request from the authors.

that setting too, the complete models are preferred to the diagonal ones. Exceptions are the diagonal GARCH-X models selected for Finland and Germany. Finally, models involving realized covariances provide better estimates, see Panel B of Table 10. In this context, the time-varying total premium is

$$\delta_{2,t} \quad = \quad GP_{2,t} + LP_{2,t} = \lambda_{W,t-1}\,\mathrm{Cov}_{t-1}[r_{I,t}, r_{W,t}] + \lambda_{I,t-1}\,\mathrm{Var}_{t-1}[r_{I,t}|r_{DP,t}]. \tag{9}$$

In Table 11, we report the summary statistics of the estimated integration degree and risk premia for the two models for each country: one with constant λ_* and one with a time-varying specification. For each country, we observe that the sample means of II_t are close between the estimated models. For what concerns the variability of the estimates of the integration index, it does not emerge a clear picture, since in 7 out 14 cases the estimates of the integration index obtained with time-varying λ are less variable than those from models with constant prices of risk. For each country, Figures 1 to 14 plot the estimated integration index for the models with $\lambda_{*,t-1}$. It is evident that the estimates of II vary across countries, in some cases in a strikingly way. This can be due to the differences in fit of each model which ultimately depends on the time series length used in estimation. Apart from that, we can recognize in the plots of the estimated II a common pattern, which essentially can be described as the occurrence of a peak in the integration process just before the financial crisis of 2008 and a subsequent decrease in the following years. Not surprisingly, the risk premia are larger when markets enter in turmoil periods, like in 2008-2009.

Further, it should be noted that the local risk premia is very small when the λ are time varying. In terms of portfolio allocation, this means that the excess returns depend only on the world factor risk. Whereas in terms of financial integration this means that the European countries are largely integrated with the World. This result is in line with the literature, as expected, which stresses that developed markets are much more integrated in the world economy than emerging markets.

	AT	BE	DK	FI	FR	GE	IE	IT	NL	NO	PT	ES	SE	UK	World
							Subsample from January 1995 to July 2007								
mean	3.562	3.918	7.766	7.321	4.120	2.264	3.376	2.727	1.854	3.311	4.087	7.273	5.199	0.007	2.587
st dev	6.109	5.638	6.153	12.013	6.172	7.546	6.071	6.896	6.383	7.150	6.698	6.958	8.032	4.373	4.614
skewness	-0.988	-1.132	-0.513	-0.495	-0.586	-0.816	-0.763	0.176	-1.119	-0.959	-0.362	-0.841	-0.508	-0.882	-0.786
kurtosis	5.282	6.129	3.540	4.405	3.794	5.413	3.674	3.796	5.505	6.053	4.318	5.831	4.015	4.352	4.267
							Subsample from August 2008 to August 2016								
mean	-12.404	-2.497	4.631	-6.731	-3.110	-1.238	-8.827	-9.752	-0.816	-3.756	-12.876	-5.755	-0.606	-0.800	0.246
st dev	9.683	7.992	6.779	7.754	6.020	6.841	8.806	7.674	6.344	7.763	6.788	7.609	6.203	4.991	5.968
skewness	-1.238	-2.228	-0.678	-0.351	-0.354	-0.626	-0.830	-0.156	-0.858	-1.400	-0.648	-0.171	-0.720	-0.479	-0.916
kurtosis	7.181	11.453	5.085	4.596	3.086	4.240	4.182	2.854	4.774	7.248	4.049	3.518	5.990	3.392	5.429
							Full sample from January 2007 to August 2016								
mean	-3.132	1.229	6.452	1.430	1.089	0.796	-1.740	-2.504	0.735	0.348	-3.024	1.811	2.765	-0.331	1.606
st dev	7.832	6.719	6.412	10.447	6.107	7.247	7.352	7.245	6.356	7.407	6.775	7.252	7.314	4.633	5.215
skewness	-1.381	-1.996	-0.603	-0.419	-0.486	-0.745	-0.932	-0.016	-1.010	-1.181	-0.478	-0.535	-0.544	-0.682	-0.903
kurtosis	8.527	11.563	4.390	4.970	3.483	5.055	4.762	3.392	5.190	6.746	4.239	4.574	4.649	3.888	5.412
JB	0.001	0.001	0.001	0.001	0.008	0.001	0.001	0.386	0.001	0.001	0.001	0.001	0.001	0.001	0.001
LB(2)	0.000	0.000	0.370	0.016	0.378	0.727	0.005	0.644	0.474	0.019	0.173	0.230	0.332	0.704	0.147
LB(4)	0.001	0.000	0.344	0.058	0.508	0.873	0.002	0.245	0.512	0.043	0.172	0.319	0.063	0.143	0.080
LB(8)	0.000	0.000	0.228	0.072	0.188	0.584	0.004	0.056	0.070	0.050	0.570	0.523	0.037	0.291	0.197
ARCH(2)	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.109	0.000	0.000	0.069	0.016	0.000	0.000	0.000
ARCH(4)	0.000	0.000	0.003	0.000	0.001	0.000	0.000	0.191	0.000	0.000	0.036	0.079	0.001	0.000	0.000
ARCH(8)	0.000	0.000	0.005	0.000	0.002	0.001	0.000	0.037	0.001	0.000	0.183	0.109	0.005	0.001	0.001
$\mathrm{Corr}(r_I, r_D)$	0.762	0.722	0.652	0.709	0.837	0.823	0.621	0.723	0.787	0.788	0.642	0.774	0.795	0.845	
$\mathrm{Corr}(r_I, r_W)$	0.710	0.689	0.652	0.627	0.822	0.790	0.598	0.689	0.776	0.751	0.605	0.738	0.729	0.845	
$\mathrm{Corr}(r_D, r_W)$	0.936	0.955	1.000	0.867	0.982	0.956	0.936	0.953	0.985	0.953	0.927	0.945	0.925	1.000	

Table 6: Summary statistics of excess returns. Monthly data on returns of European and World market indexes are in excess of the 30-day Eurodollar deposit rate. The full sample covers the period from January 1995 to August 2016. We also report some descriptive statistics for two subsamples. Mean and standard deviations are in annualized percentage terms. The test for kurtosis coefficient has been normalized to zero, JB is the Jarque-Bera test for normality based on skewness and excess kurtosis. LB(.) is the Ljung-Box test for autocorrelation of order 2, 4 and 8. ARCH(.) is the Engle's ARCH test for residual heteroscedasticity of order 2, 4 and 8. Pairwise correlations for the excess portfolio returns r_I, r_D and r_W are also reported.

Criteria	(i)		(ii)		(iii)	
	Estimate	p-value	Estimate	p-value	Estimate	p-value
			AT			
c	-0.002	0.438	-0.002	0.438	-0.002	0.438
x_3	0.383	0.000	0.383	0.000	0.383	0.000
x_8	0.511	0.000	0.511	0.000	0.511	0.000
$\bar{R}^2$	0.576		0.576		0.576	
			BE			
c	-0.001	0.586	-0.001	0.586	-0.001	0.633
x_1	0.799	0.000	0.799	0.000	0.723	0.000
x_2	-0.164	0.009	-0.164	0.009	-0.153	0.016
x_6	0.426	0.000	0.426	0.000	0.436	0.000
x_8					0.150	0.184
x_{12}					-0.120	0.085
$\bar{R}^2$	0.513		0.513		0.516	
			DK			
c	0.005	0.005	0.005	0.060	0.005	0.042
x_1	0.801	0.801	0.801	0.000	0.794	0.000
x_3					-0.210	0.012
x_4					0.324	0.057
x_7					-0.150	0.104
$\bar{R}^2$	0.422		0.422		0.436	
			FI			
c	-0.003	0.404	-0.003	0.404	-0.003	0.483
x_7	0.280	0.013	0.280	0.013	0.229	0.050
x_9	0.791	0.000	0.791	0.000	0.721	0.000
x_{10}					0.166	0.134
$\bar{R}^2$	0.500		0.500		0.503	
			FR			
c	-0.001	0.622	-0.001	0.694	0.000	0.885
x_1	1.294	0.000	1.323	0.000	1.396	0.000
x_2			-0.083	0.097	-0.082	0.101
x_3	-0.165	0.002	-0.121	0.044	-0.137	0.024
x_7					-0.099	0.119
x_{12}	-0.132	0.005	-0.139	0.003	-0.135	0.004
$\bar{R}^2$	0.698		0.700		0.702	

Table 7: Diversification Portfolio estimation results. The Table reports the estimated coefficients and the corresponding p-values from a stepwise regression procedure with forward and backward threshold criteria to obtain a DIV portfolio for each country. The threshold criteria are: (i) p-value enter < 0.05 and p-value remove > 0.10, (ii) p-value enter < 0.10 and p-value remove > 0.15, (iii) p-value enter < 0.15 and p-value remove > 0.20. The table reports also the R-squared adjusted.

Criteria	(i)		(ii)		(iii)	
	Estimate	p-value	Estimate	p-value	Estimate	p-value
			GE			
c	-0.001	0.754	-0.001	0.754	-0.001	0.754
x_5	0.537	0.000	0.537	0.000	0.537	0.000
x_8	0.329	0.000	0.329	0.000	0.329	0.000
x_9	0.152	0.005	0.152	0.005	0.152	0.005
x_{10}	0.154	0.014	0.154	0.014	0.154	0.014
x_{12}	-0.197	0.001	-0.197	0.001	-0.197	0.001
$\bar{R}^2$	0.672		0.672		0.672	
			IE			
c	-0.003	0.398	-0.002	0.491	-0.002	0.491
x_5	0.790	0.000	0.638	0.000	0.638	0.000
x_8			0.269	0.016	0.269	0.016
x_{12}			-0.141	0.092	-0.141	0.092
$\bar{R}^2$	0.383		0.393		0.393	
			IT			
c	-0.003	0.218	-0.003	0.218	-0.003	0.334
x_1	1.076	0.000	1.076	0.000	1.157	0.000
x_3	-0.211	0.010	-0.211	0.010	-0.233	0.005
x_7					-0.142	0.145
x_8	0.328	0.006	0.328	0.006	0.351	0.004
x_{12}	-0.264	0.001	-0.264	0.001	-0.264	0.001
$\bar{R}^2$	0.516		0.516		0.519	
			NL			
c	-0.001	0.799	-0.002	0.414	-0.002	0.393
x_1	0.948	0.000	0.718	0.000	0.703	0.000
x_3					-0.127	0.066
x_4					0.212	0.139
x_5			0.244	0.048	0.187	0.141
x_6			0.214	0.016	0.199	0.024
x_{12}			-0.137	0.014	-0.130	0.022
$\bar{R}^2$	0.600		0.614		0.617	
			NO			
c	0.000	0.941	0.000	0.941	0.000	0.941
x_1	0.766	0.000	0.766	0.000	0.766	0.000
x_2	0.222	0.001	0.222	0.001	0.222	0.001
x_3	0.193	0.019	0.193	0.019	0.193	0.019
x_7	-0.209	0.018	-0.209	0.018	-0.209	0.018
$\bar{R}^2$	0.614		0.614		0.614	

Table 8: Diversification Portfolio estimation results. The Table reports the estimated coefficients and the corresponding p-values from a stepwise regression procedure with forward and backward threshold criteria to obtain a DIV portfolio for each country. The threshold criteria are: (i) p-value enter < 0.05 and p-value remove > 0.10, (ii) p-value enter < 0.10 and p-value remove > 0.15, (iii) p-value enter < 0.15 and p-value remove > 0.20. The table reports also the R-squared adjusted.

Criteria	(i)		(ii)		(iii)	
	Estimate	p-value	Estimate	p-value	Estimate	p-value
			PT			
c	-0.001	0.587	-0.002	0.461	-0.003	0.339
x_1			0.541	0.078	0.488	0.114
x_4			-0.330	0.073	-0.323	0.079
x_6					0.147	0.196
x_8	0.330	0.000	0.254	0.038	0.222	0.075
x_{10}	0.432	0.000	0.330	0.001	0.336	0.000
$\bar{R}^2$	0.411		0.414		0.416	
			ES			
c	0.002	0.356	0.002	0.536	0.002	0.536
x_1			0.359	0.106	0.359	0.106
x_3			-0.146	0.060	-0.146	0.060
x_8	0.523	0.000	0.437	0.000	0.437	0.000
x_{10}	0.440	0.000	0.356	0.000	0.356	0.000
$\bar{R}^2$	0.597		0.600		0.600	
			SE			
c	0.001	0.641	0.001	0.585	0.001	0.585
x_2			-0.102	0.095	-0.102	0.095
x_4	0.351	0.005	0.436	0.001	0.436	0.001
x_5	0.258	0.053	0.228	0.089	0.228	0.089
x_9	0.232	0.000	0.233	0.000	0.233	0.000
x_{10}	0.262	0.000	0.263	0.000	0.263	0.000
x_{11}	-0.266	0.002	-0.226	0.013	-0.226	0.013
$\bar{R}^2$	0.625		0.628		0.628	
			UK			
c	-0.001	0.587	-0.001	0.413	-0.001	0.413
x_1	0.752	0.000	0.700	0.000	0.700	0.000
x_6			0.095	0.072	0.095	0.072
$\bar{R}^2$	0.713		0.715		0.715	

Table 9: Diversification Portfolio estimation results. The Table reports the estimated coefficients and the corresponding p-values from a stepwise regression procedure with forward and backward threshold criteria to obtain a DIV portfolio for each country. The threshold criteria are: (i) p-value enter < 0.05 and p-value remove > 0.10, (ii) p-value enter < 0.10 and p-value remove > 0.15, (iii) p-value enter < 0.15 and p-value remove > 0.20. The table reports also the R-squared adjusted.

Panel A: Models with time-invariant prices of risk λ_*								
	Mod. (3) diag GARCH		Mod. (7) diag GARCH-X		Mod. (4) GARCH		Mod. (8) GARCH-X	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
AT	-3171.48	-3078.91	-3161.45	-3068.87	**-3238.67**	**-3146.09**	-3230.91	-3138.33
DE	-3415.49	-3322.91	-3405.65	-3313.08	**-3472.68**	**-3380.10**	-3471.99	-3379.42
DK	-3715.15	-3622.57	-3718.45	-3625.88	-3758.30	-3665.73	**-3775.56**	**-3682.98**
FI	-2743.42	-2650.84	-2719.48	-2626.90	**-2804.76**	**-2712.19**	-2760.65	-2668.08
FR	-3706.64	-3614.06	-3694.62	-3602.04	-3738.34	-3645.76	**-3740.66**	**-3648.08**
GE	-3349.91	-3257.33	-3338.44	-3245.87	-3394.10	-3301.52	**-3398.59**	**-3306.01**
IE	-3262.49	-3169.91	-3272.28	-3179.71	-3318.41	-3225.83	**-3344.37**	**-3251.79**
IT	-3265.39	-3172.81	-3258.45	-3165.87	**-3310.80**	**-3218.22**	-3310.07	-3217.49
NL	-3681.66	-3589.08	-3669.88	-3577.30	-3709.72	-3617.14	**-3731.29**	**-3638.71**
NO	-3291.08	-3198.51	-3280.10	-3187.53	**-3328.54**	**-3235.96**	-3311.57	-3218.99
PT	-3269.06	-3176.48	-3253.02	-3160.45	-3309.17	-3216.60	**-3309.85**	**-3217.27**
ES	-3285.15	-3192.58	-3278.72	-3186.14	-3320.00	-3227.43	**-3330.67**	**-3238.09**
SE	-3222.12	-3129.54	-3180.07	-3087.49	**-3255.79**	**-3163.21**	-3246.24	-3153.66
UK	-4158.23	-4065.66	-4147.14	-4054.57	-4177.95	-4085.37	**-4200.97**	**-4108.40**

Panel B: Models with time-varying prices of risk $\lambda_{*,t}$								
	Mod. (5) diag GARCH		Mod. (9) diag GARCH-X		Mod. (6) GARCH		Mod. (10) GARCH-X	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
AT	-3118.52	-3018.93	-3171.51	-3071.92	**-3181.36**	**-3081.77**	-3162.38	-3062.79
DE	-3357.00	-3257.40	-3408.89	-3309.30	-3415.65	-3316.06	**-3420.74**	**-3321.15**
DK	-3698.79	-3599.20	-3697.12	-3597.53	-3728.05	-3628.45	**-3736.49**	**-3636.90**
FI	-2665.45	-2565.86	**-2767.26**	**-2667.67**	-2724.54	-2624.95	-2745.95	-2646.35
FR	-3685.17	-3585.58	-3671.25	-3571.66	-3694.16	-3594.57	**-3702.30**	**-3602.70**
GE	-3326.23	-3226.64	**-3354.90**	**-3255.31**	-3350.73	-3251.14	-3347.33	-3247.74
IE	-3233.27	-3133.68	-3269.40	-3169.81	-3263.2	-3163.61	**-3300.59**	**-3201.00**
IT	-3230.17	-3130.58	-3252.63	-3153.04	-3263.80	-3164.21	**-3271.12**	**-3171.53**
NL	-3657.12	-3557.53	-3644.05	-3544.46	-3682.82	-3583.23	**-3684.97**	**-3585.38**
NO	-3268.24	-3168.65	-3268.45	-3168.86	**-3282.66**	**-3183.07**	-3276.92	-3177.33
PT	-3225.54	-3125.95	-3265.52	-3165.93	-3237.11	-3137.52	**-3272.18**	**-3172.58**
ES	-3250.79	-3151.20	-3278.55	-3178.96	-3282.89	-3183.30	**-3304.00**	**-3204.41**
SE	-3156.79	-3057.20	-3207.88	-3108.29	-3177.82	-3078.23	**-3210.28**	**-3110.69**
UK	-4135.80	-4036.21	-4118.12	-4018.53	-4150.60	-4051.00	**-4157.67**	**-4058.08**

Table 10: AIC and BIC criteria. Panel A reports the values of AIC and BIC criteria of models that assumes time-invariant prices of risk λ_* (i.e. models (3), (4), (7) and (8)). Panel B reports the values of AIC and BIC criteria of models that assumes time-varying prices of risk $\lambda_{*,t}$ (i.e. models (5), (6), (9) and (10)). For each country, we select in bold the model that minimize the criterion.

Country	Model	$\widehat{\overline{II}}_t$	$sd(\widehat{II}_t)$	$\widehat{\overline{\delta}}_{1,t}$	$\widehat{\overline{GP}}_{1,t}$	$\widehat{\overline{LP}}_{1,t}$	$\widehat{\overline{\delta}}_{2,t}$	$\widehat{\overline{GP}}_{2,t}$	$\widehat{\overline{LP}}_{2,t}$
AT	4	0.503	0.155	6.523	4.491	2.032			
	6	0.538	0.095				1.020	1.020	0.000
BE	4	0.451	0.130	3.108	3.108	0.000			
	10	0.494	0.079				1.590	1.590	0.000
DK	8	0.415	0.061	1.358	1.358	0.000			
	10	0.433	0.107				1.493	1.493	0.000
FI	4	0.442	0.144	5.413	5.413	0.000			
	9	0.472	0.126				2.168	2.168	0.000
FR	8	0.662	0.097	1.625	1.625	0.000			
	10	0.662	0.120				1.787	1.735	0.052
GE	8	0.593	0.162	1.904	1.904	0.000			
	9	0.620	0.082				1.949	1.949	0.000
IE	8	0.358	0.098	1.463	1.463	0.000			
	10	0.390	0.110				1.665	1.665	0.000
IT	4	0.474	0.197	3.744	3.744	0.000			
	10	0.470	0.110				1.799	1.799	0.000
NL	8	0.570	0.121	1.540	1.540	0.000			
	10	0.577	0.129				1.675	1.670	0.005
NO	4	0.586	0.148	2.335	2.335	0.000			
	6	0.641	0.111				1.661	1.661	0.000
PT	8	0.373	0.122	1.295	1.295	0.000			
	9	0.377	0.145				1.391	1.391	0.000
ES	8	0.535	0.114	1.743	1.743	0.000			
	10	0.533	0.154				1.963	1.959	0.003
SE	4	0.599	0.128	6.361	4.231	2.130			
	9	0.601	0.116				1.763	1.763	0.000
UK	8	0.666	0.105	1.233	1.233	0.000			
	10	0.661	0.128				1.312	1.307	0.005

Table 11: Summary statistics of integration index and risk premia. For each European country and for each selected model, the table reports the average estimated integration index and its standard deviation, denoted by $\widehat{\overline{II}}_t$ and $sd(\widehat{II}_t)$, respectively. Moreover, for the models estimated assuming time-invariant (time-varying) prices of risk, we report the average estimates of total $\widehat{\overline{\delta}}_{1,t}$ ($\widehat{\overline{\delta}}_{2,t}$), global $\widehat{\overline{GP}}_{1,t}$ ($\widehat{\overline{GP}}_{2,t}$), and local risk premium $\widehat{\overline{LP}}_{1,t}$ ($\widehat{\overline{LP}}_{2,t}$).

Figure 1: II and total risk premium of Austria (AT). The first panel shows the integration degree estimated over the model (6). The second panel plots the corresponding estimates of total risk premium.

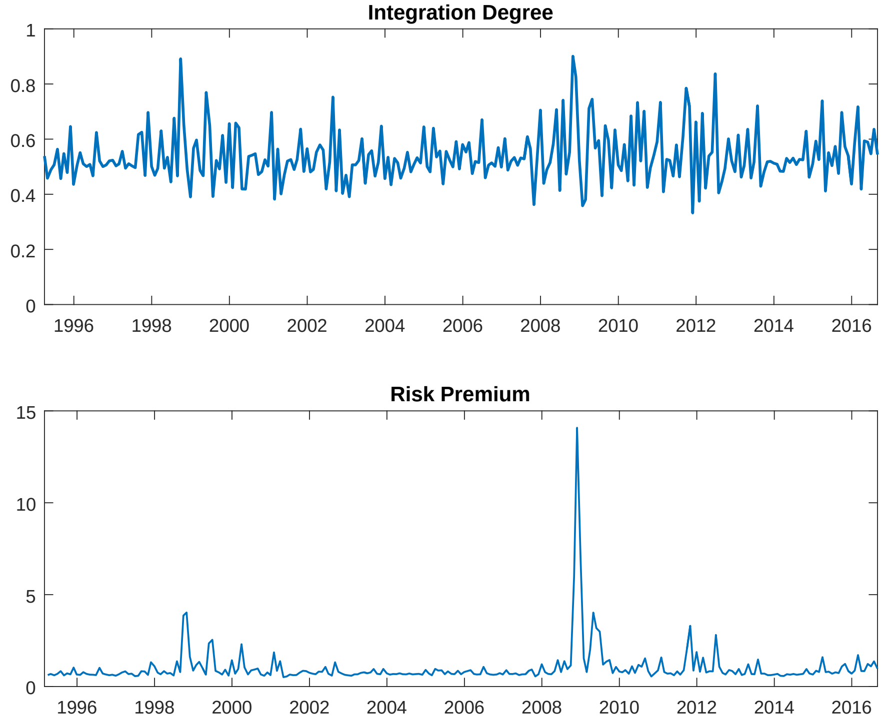


Figure 2: II and total risk premium of Belgium (BE). The first panel shows the integration degree estimated over the model (10). The second panel plots the corresponding estimates of total risk premium.

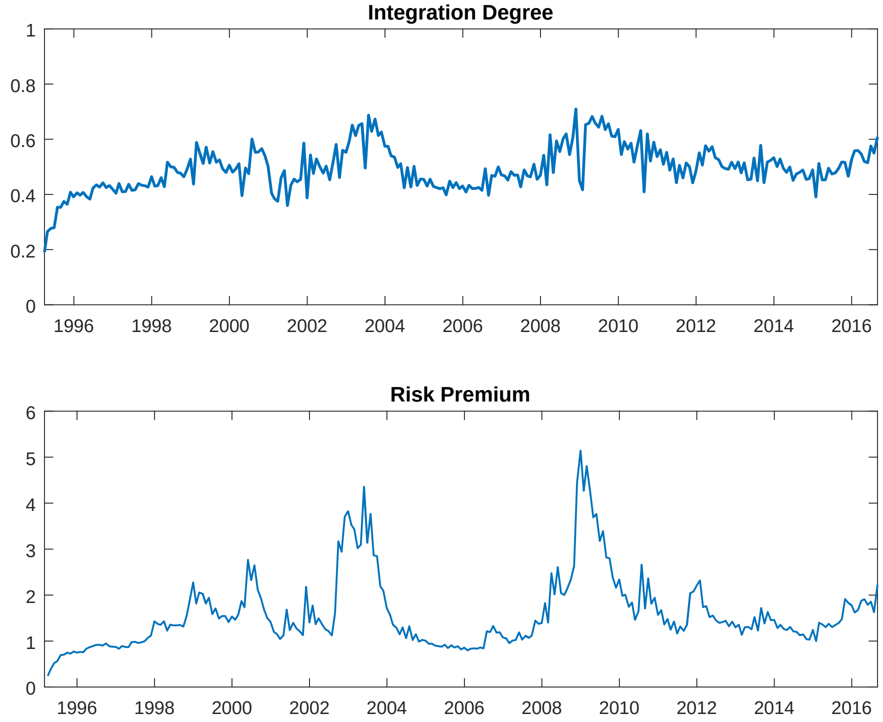


Figure 3: II and total risk premium of Denmark (DE). The first panel shows the integration degree estimated over the model (10). The second panel plots the corresponding estimates of total risk premium.

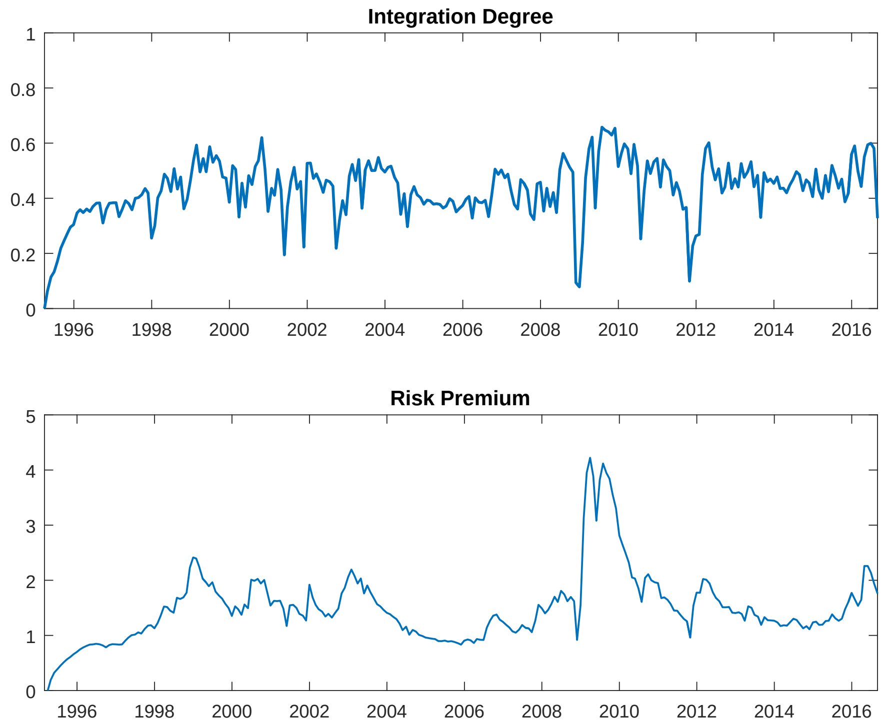


Figure 4: II and total risk premium of Finland (FI). The first panel shows the integration degree estimated over the model (9). The second panel plots the corresponding estimates of total risk premium.

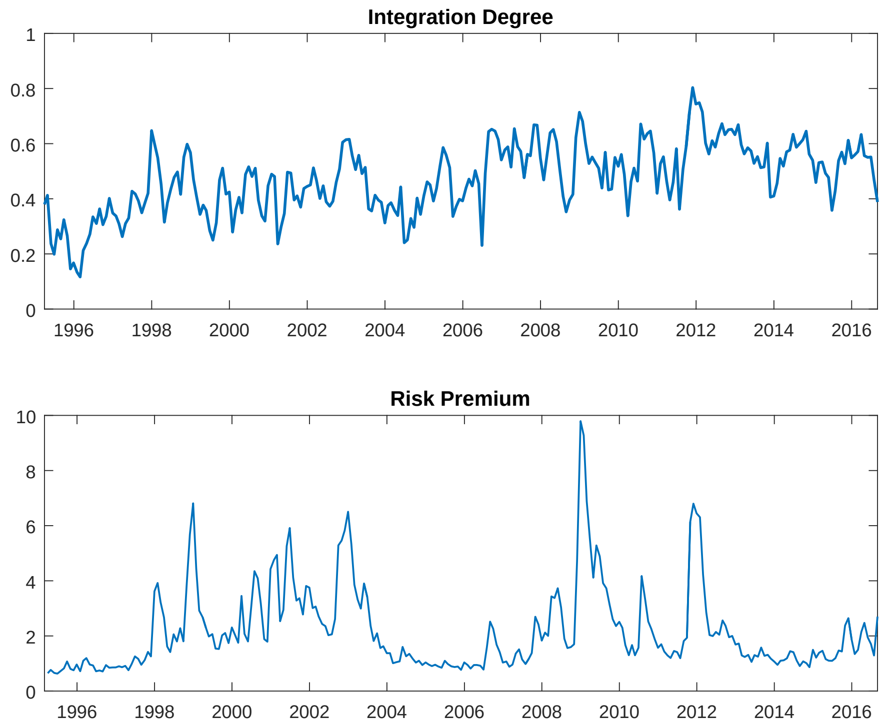


Figure 5: II and total risk premium of France (FR). The first panel shows the integration degree estimated over the model (10). The second panel plots the corresponding estimates of total risk premium.

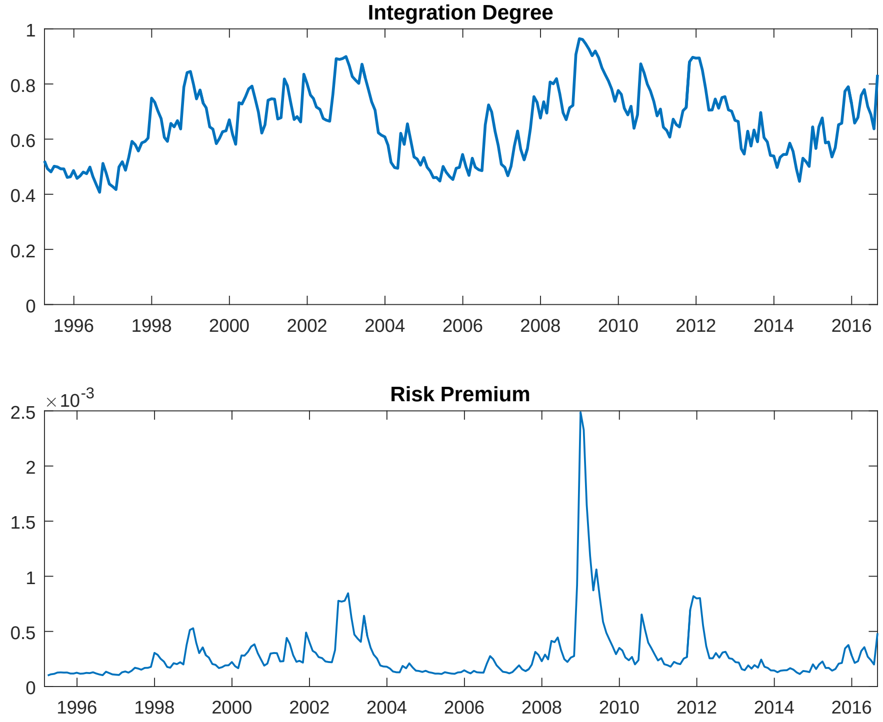


Figure 6: II and total risk premium of Germany (GE). The first panel shows the integration degree estimated over the model (9). The second panel plots the corresponding estimates of total risk premium.

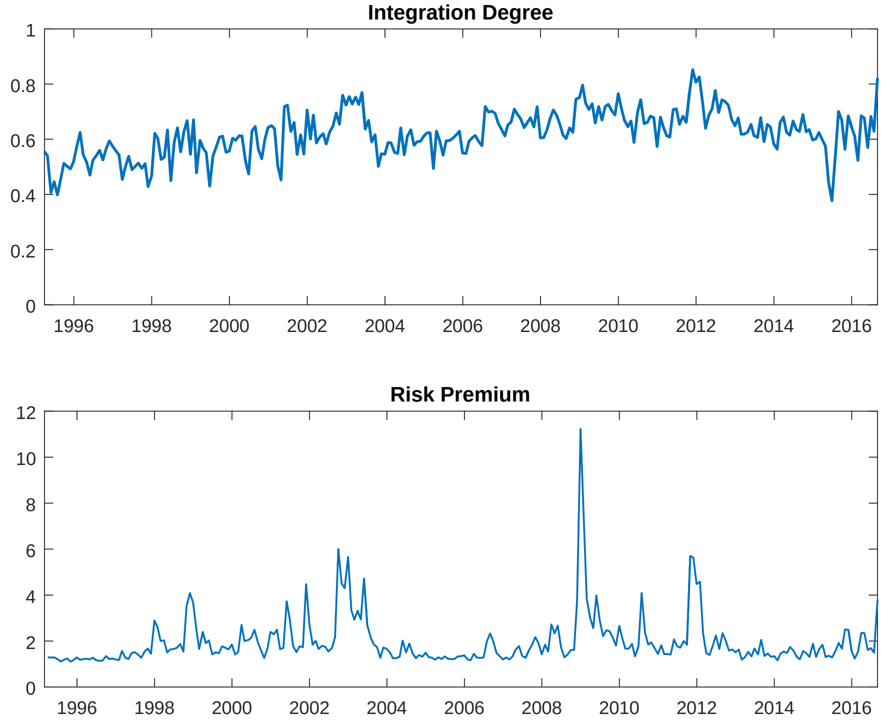


Figure 7: II and total risk premium of Ireland (IE). The first panel shows the integration degree estimated over the model (10). The second panel plots the corresponding estimates of total risk premium.

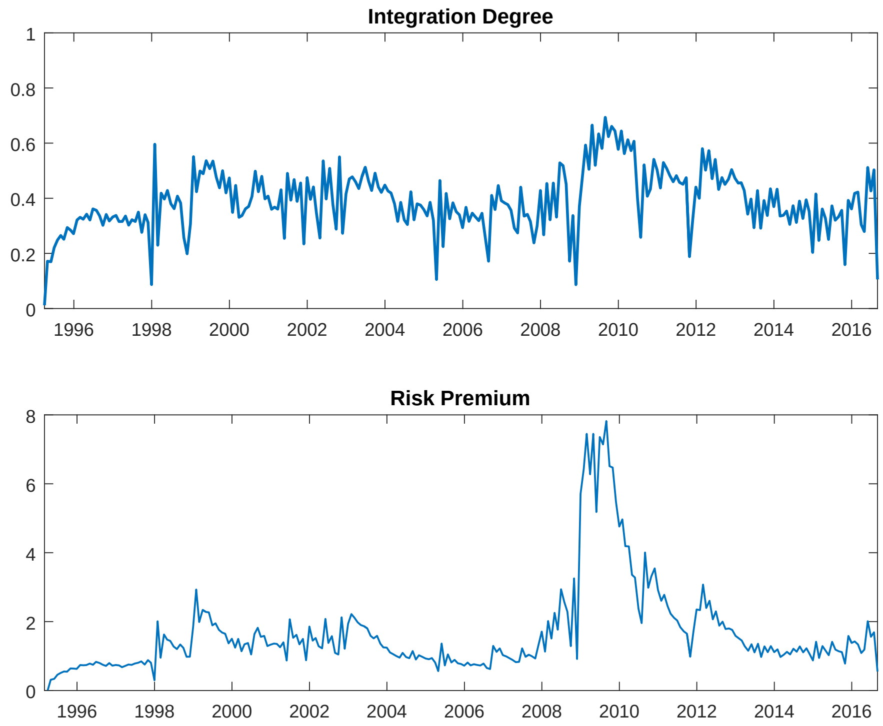


Figure 8: II and total risk premium of Italy (IT). The first panel shows the integration degree estimated over the model (10). The second panel plots the corresponding estimates of total risk premium.

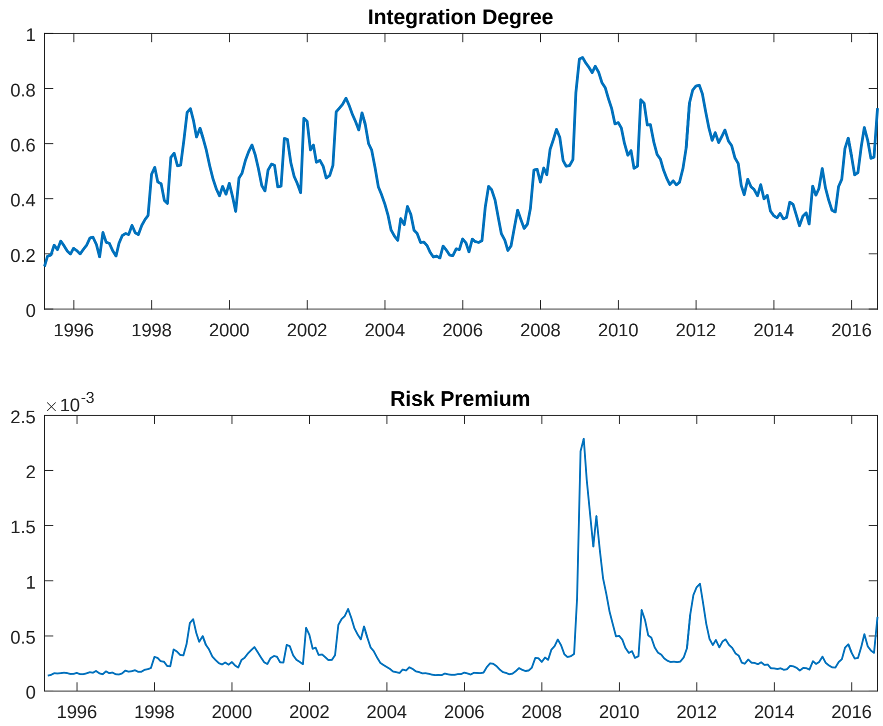


Figure 9: II and total risk premium of Netherland (NL). The first panel shows the integration degree estimated over the model (10). The second panel plots the corresponding estimates of total risk premium.

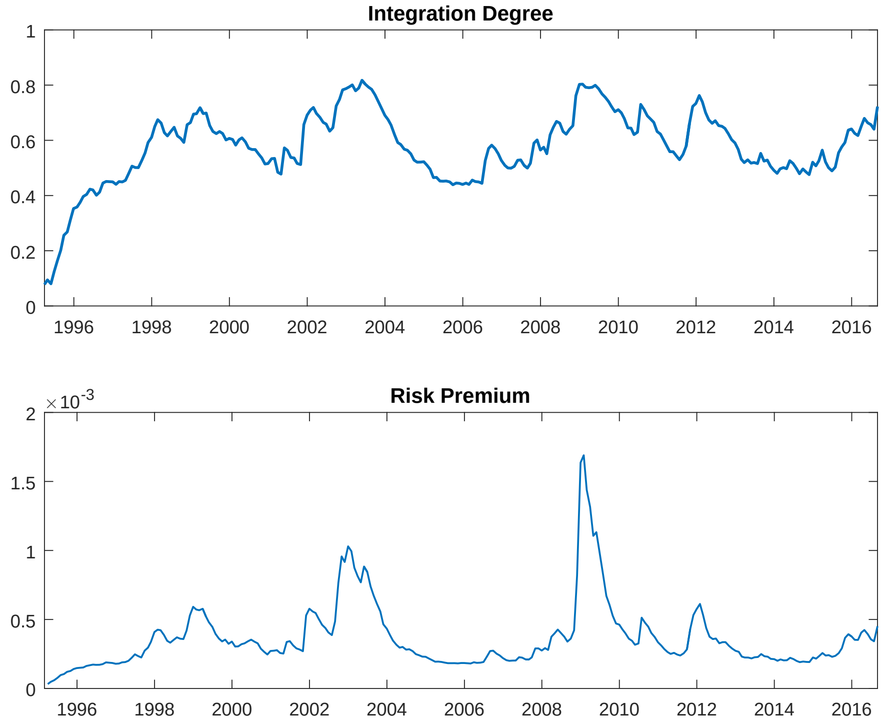


Figure 10: II and total risk premium of Norway (NO). The first panel shows the integration degree estimated over the model (6). The second panel plots the corresponding estimates of total risk premium.

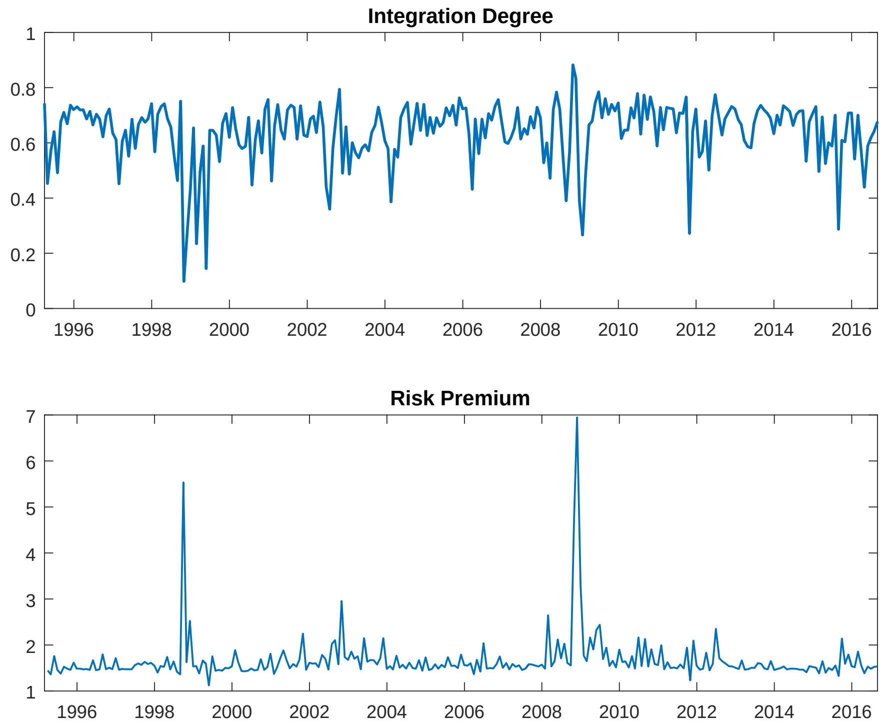


Figure 11: II and total risk premium of Portugal (PT). The first panel shows the integration degree estimated over the model (10). The second panel plots the corresponding estimates of total risk premium.

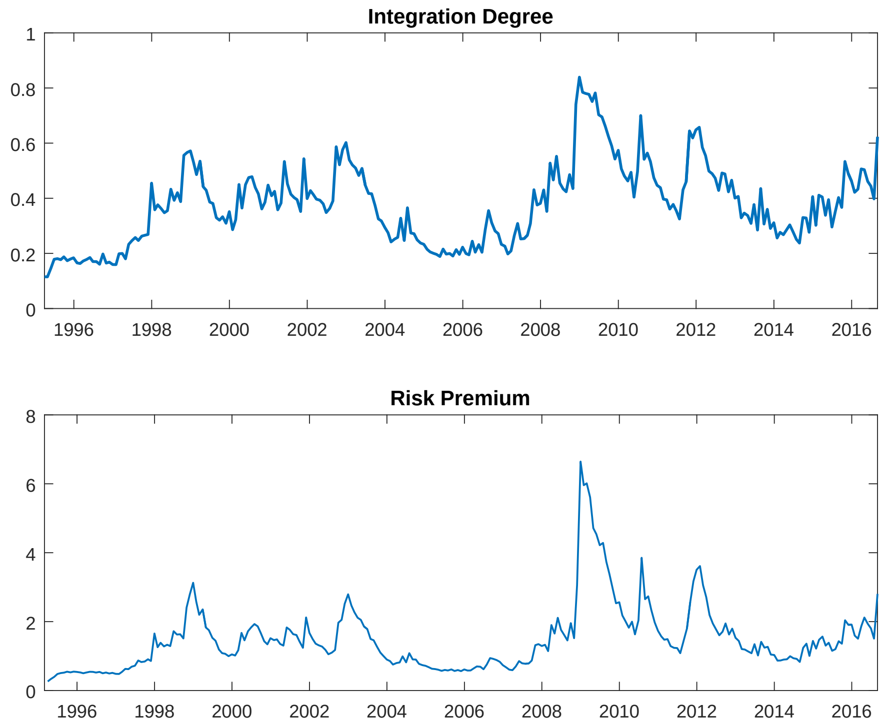


Figure 12: II and total risk premium of Spain (ES). The first panel shows the integration degree estimated over the model (10). The second panel plots the corresponding estimates of total risk premium.

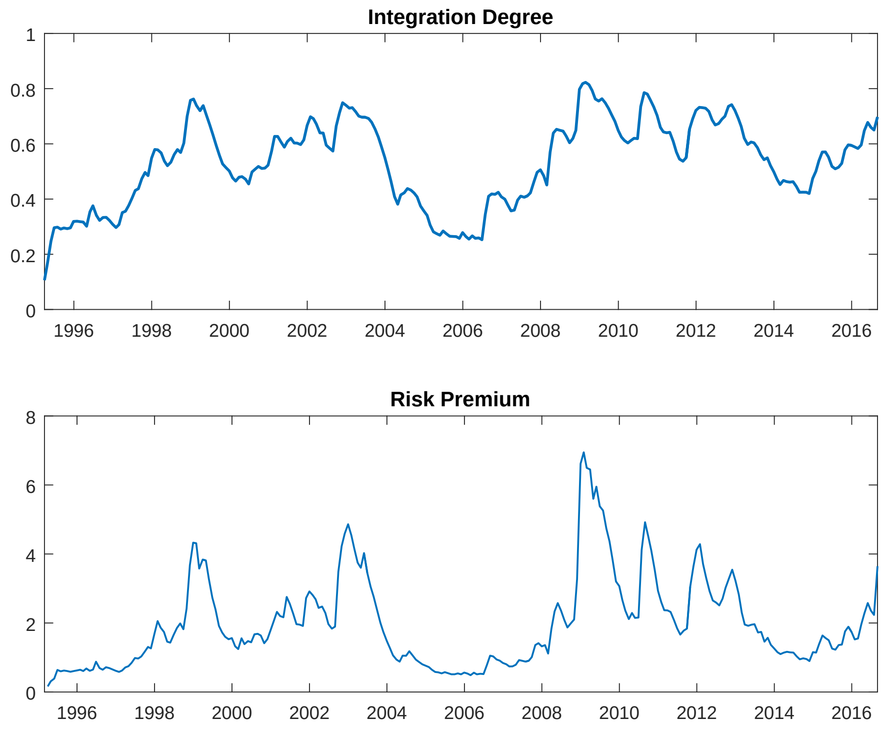


Figure 13: II and total risk premium of Sweden (SE). The first panel shows the integration degree estimated over the model (10). The second panel plots the corresponding estimates of total risk premium.

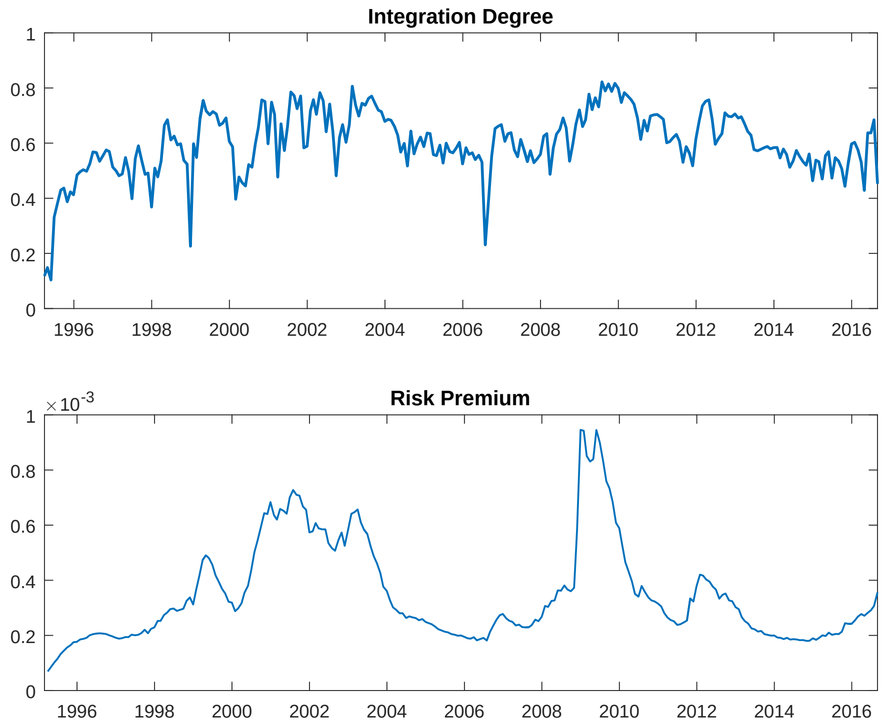
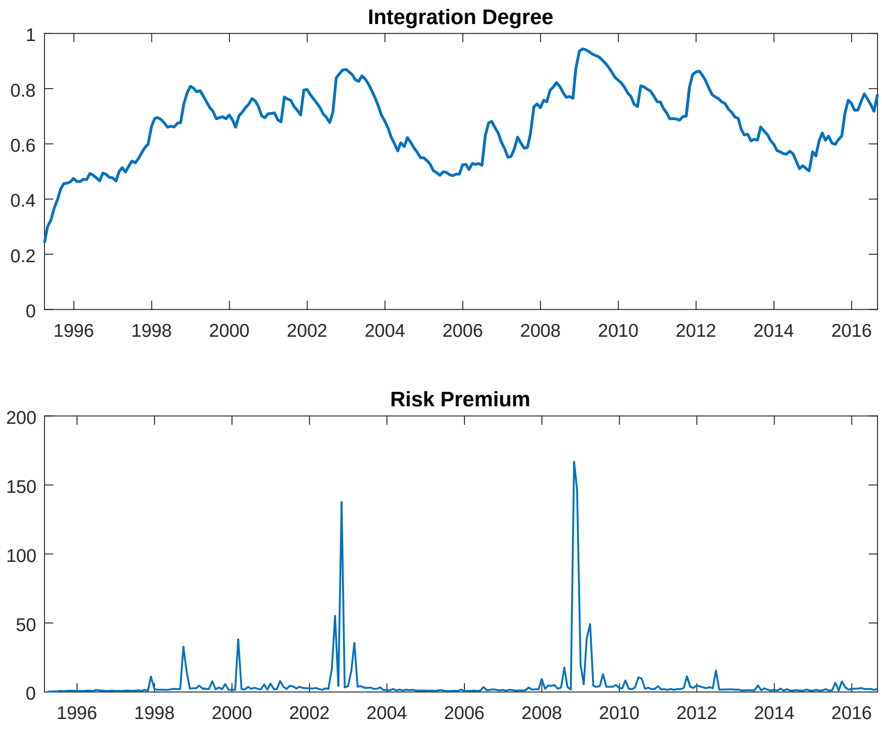


Figure 14: II and total risk premium of United Kingdom (UK). The first panel shows the integration degree estimated over the model (10). The second panel plots the corresponding estimates of total risk premium.



6. Conclusions

Using a conditional version of the IAPM by Errunza and Losq (1985) we investigate the integration of a group of European stock markets in the World market. Differently from the current literature on the topic we specify alternative econometric models for the conditional covariance of stock indexes which include as a measure of past variability the monthly realized covariances. This makes our analysis more robust to potential model misspecifications. We consider also alternative specifications for the prices of risk, global and local, and we conclude that estimate of the integration index with the constant and time varying parametrizations are very close. We show that the estimated time-varying integration index is stable across the sample period with the exception of the financial crisis period. The local risk factor which is one of the factors in determining the excess return in case of mild segmentation does not seem to be a determinant factor in the European markets, in the sample period considered. Furthermore, the financial integration measures obtained with the estimation of the conditional IAPM can be carefully studied in order to better understand the determinants and the dynamic behavior.

A Models of conditional covariances

The variances and covariances of stock returns vary over time. As a result, many important financial applications require a model of the conditional covariance matrix. Three distinct categories of methods for estimating a latent conditional covariance matrix have evolved in the literature. In the first category are the various forms of the multivariate GARCH model where forecasts of future volatility depend on past volatility and shocks, see Silvennoinen and Terasvirta (2009). In the second category, authors have modeled asset return variances and covariances as functions of a number of predetermined variables (e.g. Ferson (1995)). The third category includes multivariate stochastic volatility models (e.g. Chib et al. (2009)).

The model in Eq. (6) can be written as

$$r_t = \Lambda_{t-1} F_t + \varepsilon_t \tag{10}$$

where $F_t' = \begin{bmatrix} H_{I,W,t} & H_{I,t}(1-\rho_t^2(I,DP)) & H_{DP,W,t} & H_{W,t} \end{bmatrix}$, and H_t is the conditional variance of $r_t = [r_{W,t}, r_{DP,t}, r_{I,t}]'$, i.e. $E_{t-1}[\varepsilon_t \varepsilon_t']$. The conditional expectation $E_{t-1}[\cdot] = E[\cdot|\Phi_{t-1}]$, is taken w.r.t. Φ_{t-1} that is by assumption the σ-field generated by past values of observable variables. The conditional and unconditional moments of ε_t are

$$\begin{aligned} E_{t-1}(\varepsilon_t) &= 0 \\ E_{t-1}(\varepsilon_t \varepsilon_t') &= H_t \\ E(\varepsilon_t \varepsilon_t') &= \Sigma. \end{aligned}$$

The assumption in GARCH modeling is, when we have N assets (or portfolios) in the system, that H_t is a matrix $(N \times N)$ positive definite and measurable with respect to the information set $\Phi_{t-1} = \{\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots\}$. The correlation matrix:

$$\mathrm{Corr}_{t-1}(\varepsilon_t) = R_t = D_t^{-1/2} H_t D_t^{-1/2}$$

$$D_t = \mathrm{diag}(h_{11,t}, \ldots, h_{NN,t})$$

N assets: N variances + $\frac{1}{2}N(N-1)$ covariances = $\frac{N}{2}(N+1)$.
Two alternative approaches:

- Models of H_t
- Models of D_t and R_t

The parametrization of H_t as a multivariate GARCH, which means as a function of the information set Φ_{t-1}, allows each element of H_t to depend on *q* lagged of the squares and cross-products of ϵ_t, as well as *p* lagged values of the elements of H_t. So the elements of the covariance matrix follow a vector of ARMA process in squares and cross-products of the disturbances. Any candidate model should satisfy the following conditions:

1. Diagonal elements of H_t must be strictly positive;
2. Positive definiteness of H_t;
3. Stationarity: $E[H_t]$ exists, finite and constant w.r.t. t.

Furthermore, the estimation procedure should be flexible for increasing N and it should allow for covariance spillovers and feedbacks.It is desirable that the coefficients could have an economic or financial interpretation. In the literature we can identify three approaches for constructing multivariate GARCH models:

1. direct generalizations of the univariate GARCH model of Bollerslev (1986): VEC, BEKK and factor models;
2. linear combinations of univariate GARCH models: (generalized) orthogonal models and latent factor models;
3. nonlinear combinations of univariate GARCH models: constant and dynamic conditional correlation models (CCC and DCC), copula-GARCH models;

When elements of the conditional covariance matrix enter into the conditional mean, like in Eq. (10), the model is called GARCH-in-mean. The numerical optimization of the sample log-likelihood function becomes more involved.

In the following, we present few alternative parameterizations for the conditional covariance matrix.

- Engle and Kroner (1995) propose a parametrization (*BEKK*) that imposes positive definiteness restrictions:

$$H_t = CC' + \sum_{k=1}^{K}\sum_{i=1}^{q} A_{ik}\varepsilon_{t-i}\varepsilon'_{t-i}A'_{ik} + \sum_{k=1}^{K}\sum_{i=1}^{p} B_{ik}H_{t-i}B'_{ik} \tag{11}$$

  where C, A_{ik} and B_{ik} are $(N \times N)$.

  - The intercept matrix is decomposed into CC', where C is a lower triangular matrix.
  - Without any further assumption CC' is positive semidefinite.
  - This representation is general, it includes all positive definite diagonal representations and nearly all positive definite *vech* representations.

  For exposition simplicity we will assume that $p = q = K = 1$, i.e. GARCH(1,1) model:

$$H_t = CC' + A\varepsilon_{t-1}\varepsilon'_{t-1}A' + BH_{t-1}B'. \tag{12}$$

  In general, this parametrization guarantees that when C and H_0 are full rank matrices the H_t is positive definite with probability one. Further, when $K = 1$ and the diagonal elements in C, a_{11} and b_{11} are restricted to be positive then there exists no other C, A, B in the model (12) that will give an equivalent representation. The log-likelihood function for $\{\varepsilon_T, \ldots, \varepsilon_1\}$ obtained under the assumption of conditional multivariate normality, i.e. $\varepsilon_t|\Phi_{t-1} \sim N(0, H_t)$, is:

$$\log L_T(\varepsilon_T, \ldots, \varepsilon_1; \theta) = -\frac{1}{2}\left[TN\log(2\pi) + \sum_{t=1}^{T}\left(\log|H_t| + \varepsilon'_t H_t^{-1}\varepsilon_t\right)\right],$$

  where $\theta = (\theta'_r, \theta'_H)'$ is the parameter vector written as

$$\begin{aligned}\theta_r &= (\gamma_I, \gamma_W)'\\ \theta_H &= (\text{vech}(C)', \text{vec}(A)', \text{vec}(B)')',\end{aligned}$$

  where vech stacks the columns from the principal diagonal downwards in a column vector whereas the vec operator transforms a matrix in a column vector by stacking the columns of the matrix one underneath the other. It should be noted that the assumption of conditional normality can be quite restrictive. The symmetry imposed under normality is difficult to justify, and the tails of even conditional distributions often seem fatter than that of normal distribution.

- In the spirit of the GARCH-X model (see Engle, 2002a), let $\varepsilon_t = H_t^{1/2}z_t$

$$E[\varepsilon_t\varepsilon'_t|\mathcal{F}_{t-1}^{HF}] = H_t$$

  $\mathcal{F}_t^{HF}$ is the information set generated by returns sampled at a higher frequency than that of r_t. H_t is the conditional covariance of r_t. The dynamics of H_t is specified as GARCH(1,1)-X in BEKK form

$$H_t = C_H C'_H + A_H RC_{t-1}A'_H + B_H H_{t-1}B'_H \tag{13}$$

  C_H is a (3×3) lower triangular matrix with 6 parameters. Assuming that H_0 and RC_0 are positive definite, the parameterization in Eq. (13) guarantees that H_t is positive semidefinite for all t given that H_0 and M_0 are positive semidefinite. If C_H is a full rank matrix, then H_t is a positive definite for all t. The Gaussian log-likelihood results to be

$$\ell(\theta) = \sum_t \log f(r_t|\mathcal{F}_{t-1}) = -\frac{3}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\Big[(r_t - \Lambda F_t)'H_t^{-1}(r_t - \Lambda F_t) + \log|H_t|\Big]$$

- A natural restriction of both models above is to consider A and B in Eq. (12) and A_H and B_H in Eq. (13) diagonal. For instance, consider GARCH(1,1)-BEKK, $N = 2$ with

$$A = \text{diag}(a_{11}, a_{22},) \quad B = \text{diag}(b_{11}, b_{22})$$

the model reduces to

$$\begin{aligned} H_t &= CC' + \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1t-1} & \varepsilon_{1t-1}\varepsilon_{2t-1} \\ \varepsilon_{2t-1}\varepsilon_{1t-1} & \varepsilon^2_{2t-1} \end{bmatrix} \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}' \\ &+ \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} \begin{bmatrix} h_{11t-1} & h_{12t-1} \\ h_{21t-1} & h_{22t-1} \end{bmatrix} \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix}' \end{aligned}$$

$$\begin{aligned} h_{11,t} &= c^2_{11} + a^2_{11}\epsilon^2_{1t-1} + b^2_{11}h_{11t-1} \\ h_{12,t} &= c_{21}c_{11} + a_{11}a_{22}\epsilon_{1t-1}\epsilon_{2t-1} + b_{11}b_{22}h_{12t-1} \\ h_{22,t} &= c_{21}c_{11} + c^2_{22} + a^2_{22}\epsilon^2_{1t-1} + b^2_{22}h_{11t-1} \end{aligned}$$

This model is equivalent to the Hadamard BEKK:

$$H_t = CC' + aa' \odot \varepsilon_{t-1}\varepsilon'_{t-1} + bb' \odot H_{t-1}$$

positive definiteness is not guaranteed. Positive semidefiniteness is obtained by imposing p.s.d. of all terms.

B Realized covariance

We assume that the $(n \times 1)$ vector of log-prices $X(t) = (X_1(t), \ldots, X_n(t))'$ follows a Brownian semimartingale (which belongs to the class of stochastic volatility semimartinagales) in continuous time defined on some probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), P)$

$$X(t) = \int_0^t a(u)\mathrm{d}t + \int_0^t \sigma(u)\mathrm{d}W(u) \tag{14}$$

where μ is a vector of predictable locally bounded drifts (lack of arbitrage opportunities), σ is the instantaneous or spot covolatility process whose elements are all càdlàg and W is a vector of independent Brownian motions. The *spot covariance matrix* is $\Sigma(t) = \sigma(t)\sigma(t)'$, with $\int_0^t \Sigma_{X,kk}(u)\mathrm{d}u < \infty$. The *integrated covariance matrix*

$$X[t] = \int_0^t \Sigma(u)\mathrm{d}u.$$

Using M equally spaced intra-day high-frequency returns, defined as

$$r(t, i) = X\Big((t-1) + \frac{i}{M}\Big) - X\Big((t-1) + \frac{i-1}{M}\Big), \quad i = 1, \ldots, M.$$

The realized covariation matrix of $X(t)$ over the day t

$$RC_t = \sum_{i=1}^{M} r(t, i)r(t, i)' \tag{15}$$

which is a consistent estimate of the increment of the quadratic covariation of $X(t)$

$$RC_t \xrightarrow{p} X[t] - X[t-1] = \int_{t-1}^t \Sigma(u)\mathrm{d}u = p \lim_{M\to\infty} \sum_{i=1}^{M} r(t, i)r(t, i)' \tag{16}$$

for $M \to \infty$. $\int_{t-1}^t \Sigma(u)\mathrm{d}u$ is the *actual covariance matrix* of the local martingale component of X. Under no leverage assumption, Barndorff-Nielsen and Shephard (2004) show that, as $M \to \infty$, the asymptotic law of

$$\sqrt{M}\left\{RC_t - \int_{t-1}^t \Sigma(u)\mathrm{d}u\right\} \tag{17}$$

is mixed normal with mean 0 and random covariance matrix $(n^2 \times n^2)$ Ω_t

$$\Omega_t = \left\{ \int_{t-1}^{t} \{\Sigma_{kk'}(u)\Sigma_{ll'}(u) + \Sigma_{kl'}(u) + \Sigma_{lk'}(u)\} \mathrm{d}u \right\}_{k,k',l,l'=1,\dots,n}$$

The k, k', l, l' element of Ω_t corresponds to the asymptotic covariance between the k, lth and the k', l'th elements of the matrix in Eq. (17). The stochastic matrix Ω_t is singular. It is unknown but it can be replaced by a consistent, positive semi-definite estimator. To avoid symmetric replication in the realized covariation matrix we can employ a vech transformation, i.e. $\mathrm{vech}(RC_t)$. In this case the limit theory becomes

$$\sqrt{M} \left\{ \mathrm{vech}(RC_t) - \mathrm{vech}\left(\int_{t-1}^{t} \Sigma(u)\mathrm{d}u \right) \right\} \xrightarrow{L} N(0, \Pi_t) \tag{18}$$

with $\Pi_t = L\Omega_t L'$, where L is such that $\mathrm{vech}(A) = L\mathrm{vec}(A)$ for a symmetric matrix A. With $q_{j,i} = \mathrm{vech}(r(t,i)r(t,i)')$

$$G_t = \sum_{j=1}^{M} q_{j,t}q'_{j,t} - \frac{1}{2} \sum_{j=0}^{M-1} (q_{j,t}q'_{j+1,t} + q_{j+1,t}q'_{j,t})$$

$$MG_t \xrightarrow{p} \Pi_t.$$

B1 Monthly realized covariance

The daily log-return in the *m*-th month is calculated as

$$r(t,m) = X\Big((m-1) + \frac{t}{D_m}\Big) - X\Big((m-1) + \frac{t-1}{D_m}\Big), \quad t = 1, \dots, D_m. \tag{19}$$

The quadratic variation of $X(t)$ over the month m

$$X[1] = p \lim_{n\to\infty} \sum_{j=1}^{n} r(t,m)r(t,m)' \tag{20}$$

see, e.g. (Protter, 2004, p.66-77) and (Jacod and Shiryaev, 2003, p.51). Quadratic variation is crucial to the economics of financial risk, see the reviews, for example, by Andersen et al. (2010) and Barndorff-Nielsen and Shephard (2007) and more recently Ait-Sahalia and Jacod (2014).

We use daily data to construct estimates of the monthly realized variances and covariances of stock portfolios. In principle, by using high-frequency data we obtain an estimate of the matrix of *quadratic variations and covariations* that differs from the true conditional covariance matrix by mean zero errors. This is not what happens when we use daily data to compute the monthly realized covariance matrix. For illustratory purposes, suppose the log-price follows a diffusion process with constant volatility (Black-Scholes model):

$$\mathrm{d}p(t) = \mu \mathrm{d}t + \sigma \mathrm{d}W(t) \tag{21}$$

the integrated variance for the period $[t-h, t]$ results to be:

$$IV_t = \int_{t-h}^{t} \sigma^2(u)\mathrm{d}u = \int_{0}^{h} \sigma^2(t-h+s)\mathrm{d}s = h \cdot \sigma^2.$$

The daily log-return for day t in month m is equal to

$$\begin{aligned} r(t,m) &= p\Big((m-1) + \frac{t}{D_m}\Big) - p\Big((m-1) + \frac{t-1}{D_m}\Big) \\ &= \mu\frac{1}{D_m} + \sigma\Big[W\Big((m-1) + \frac{t}{D_m}\Big) - W\Big((m-1) + \frac{t-1}{D_m}\Big)\Big], \quad t = 1, 2, \dots, D_m \end{aligned}$$

where D_m is the number of days in month m. Hence, $r(t,m)$ is $i.i.d.N(\mu, \frac{\sigma^2}{D_m})$. To estimate the integrated variance over the month, i.e.

$$IV_m = \int_{0}^{D_m} \sigma^2\Big((m-1) + \frac{s}{D_m}\Big)\mathrm{d}s = D_m \cdot \sigma^2,$$

we can employ the realized variance using daily returns in the month

$$RV_m = \sum_{t=1}^{D_m} r(t,m)^2.$$

The expected value of the RV_m

$$E\Big[RV_m\Big] = E\left[\sum_{i=1}^{D_m} r(t,m)^2\right] = \frac{\mu^2}{D_m} + \sigma^2.$$

It is evident that using daily returns we have a coarse approximation to the quadratic variation of returns, with a bias of order $O(D_m^{-1})$.

In order to get rid of the bias term in realized variance, we can use the centred returns, i.e. $\tilde{r}(t,m) = r(t,m) - \bar{r}(m)$,

$$\tilde{RV}_m = \sum_t \tilde{r}(t,m)^2$$

The asymptotics of the realized covariation has been established by Barndorff-Nielsen and Shephard (2007) under the assumption that on any bounded interval $\sigma_{X,kk}(t)$ is bounded away from zero and infinity, the processes $\int_0^t a(u)\mathrm{d}t$ and $\Sigma_X(t)$ are jointly independent of the Brownian motion W.

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